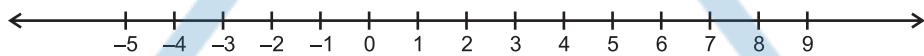


# CHAPTER-1

## NUMBER SYSTEMS

### KEY POINTS



- 1, 2, 3, ..... are natural numbers which are represented by N.
- 0, 1, 2, 3, ..... are whole numbers which are represented by W.
- ..... -3, -2, -1, 0, 1, 2, 3, ..... are Integers which are represented by Z or I.
- A number is rational number if
  - (a) it can be represented in the form of  $\frac{p}{q}$  where p and q are integers and  $q \neq 0$ .  
or  
(b) its decimal expansion is terminating (e.g.  $\frac{2}{5} = 0.4$ )  
or  
(c) its decimal expansion is non-terminating recurring (repeating)  
(e.g.  $0.\overline{1234} = 0.1\dot{2}3\dot{4} = 0.1234234\dots$ )
- A number is irrational number if
  - (a) it can not be represented in the form of  $\frac{p}{q}$  where p and q are integers and  $q \neq 0$ .  
or  
(b) its decimal expansion is non-terminating non-recurring (e.g. 0.1010010001.....)
- All rational and irrational numbers collectively form real numbers.
- There are infinite rational numbers between any two rational numbers.
- There is a unique real number corresponding to every point on the number line. Also, corresponding to each real number, there is a unique point on the number line.
- Rationalisation means to change the Irrational denominator to rational form.
- To rationalise the denominator of  $\frac{1}{\sqrt{a} + b}$ , We multiply this by  $\frac{\sqrt{a} - b}{\sqrt{a} - b}$ , where a and b are integers.

- Laws of Exponents : Let  $a > 0$  be a real number and  $m$  and  $n$  are rational numbers, then

$$1) \quad a^m \cdot a^n = a^{m+n}$$

$$2) \quad a^m \div a^n = a^{m-n}$$

$$3) \quad (a^m)^n = a^{mn}$$

$$4) \quad a^m \cdot b^m = (ab)^m$$

$$5) \quad a^0 = 1$$

$$6) \quad a^{-m} = \frac{1}{a^m}$$

- For positive real number  $a$  and  $b$ , the following Identities hold

$$1) \quad \sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$$

$$2) \quad \sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$$

$$3) \quad (\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$$

$$4) \quad (\sqrt{a} + \sqrt{b})^2 = a + 2\sqrt{ab} + b$$

$$5) \quad (a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$$

### Real Numbers

Rational Numbers  $\left(\frac{p}{q}\right)$

Terminating  
Decimal Expansion

$$q = 2^m \times 5^n$$

$$\left(\frac{2}{5} = 0.4\right)$$

Non Terminating  
Recurring Decimal  
Expansion

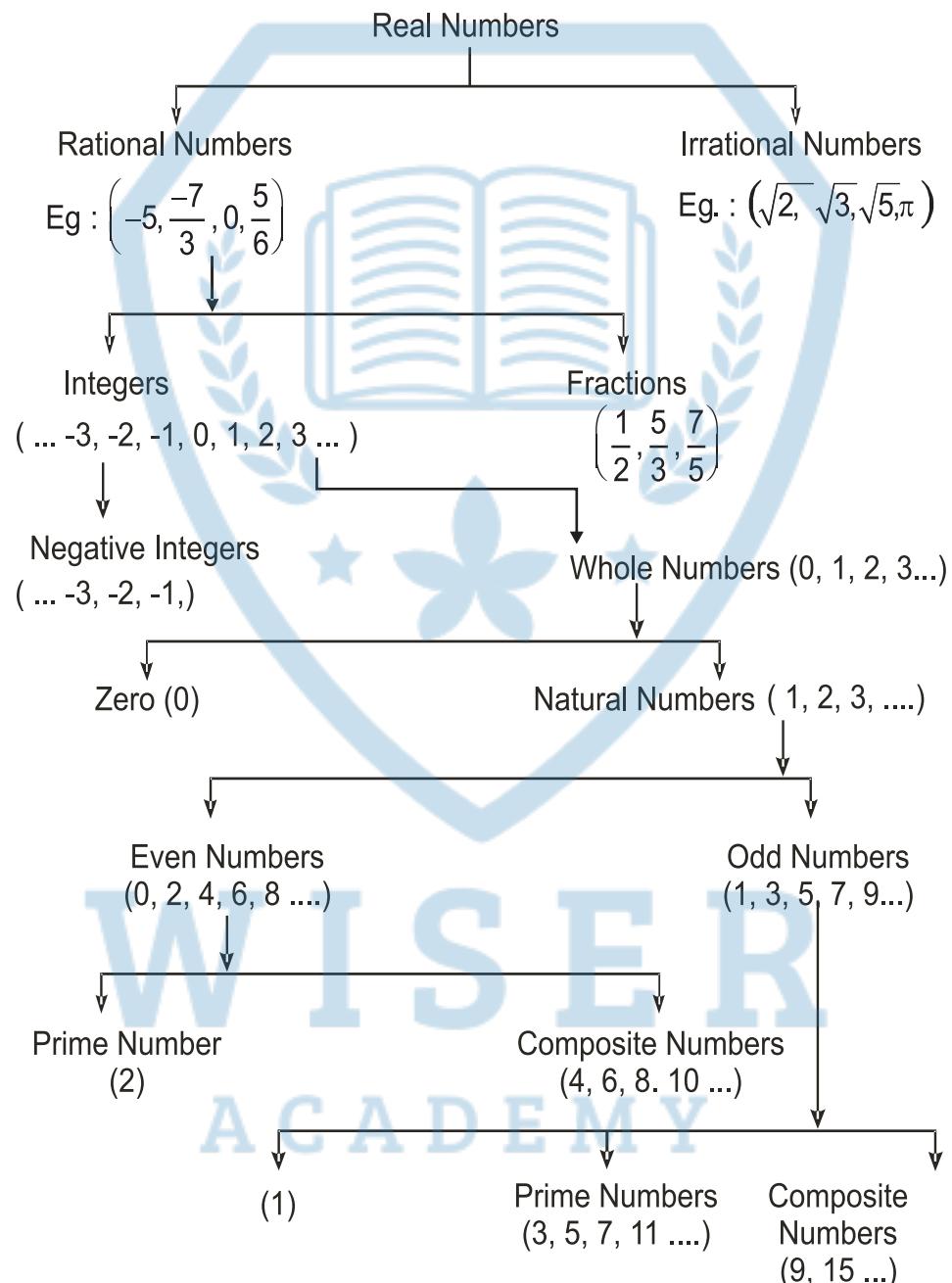
$$\left(\frac{10}{3} = 3.33\dots = 3.\bar{3}\right)$$

Non-Terminating  
Non-Recurring  
Decimal Expansion  
 $(\sqrt{2} = 1.414\dots)$

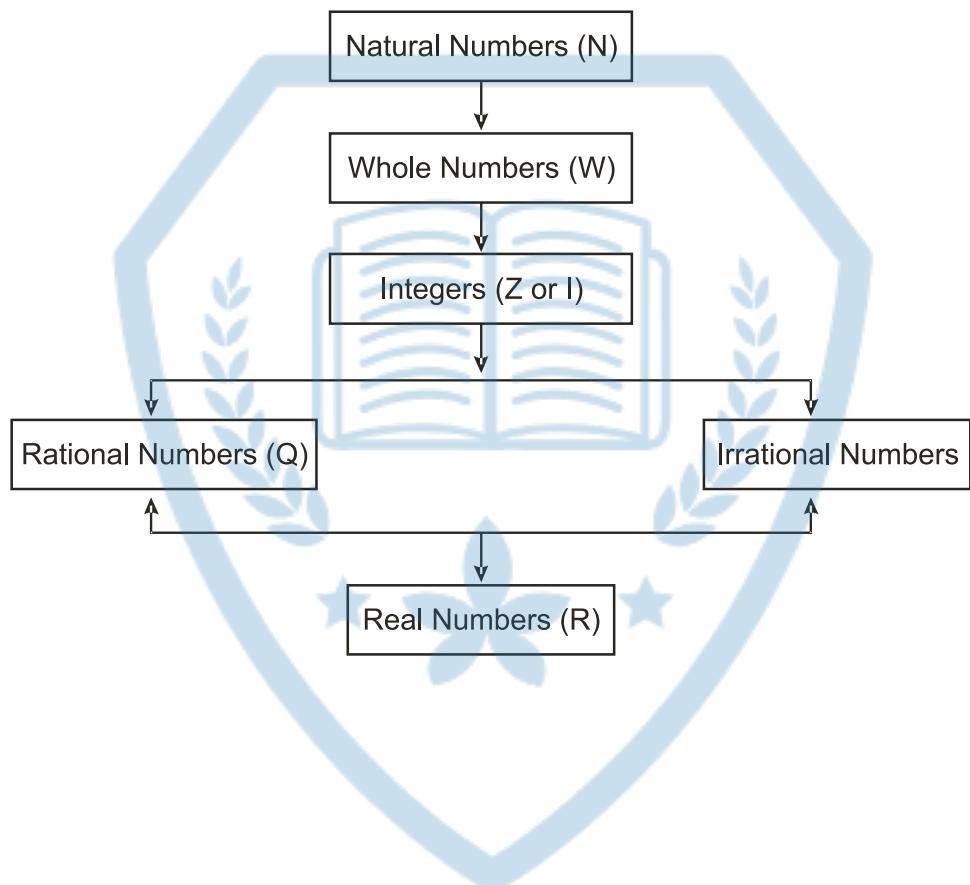
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## ACADEMY

- Types of Numbers :



- Sequence of various types of Numbers :



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### **Part (A)**

1. Write first five whole numbers in  $\frac{p}{q}$  form, where p and q are integers and  $q \neq 0$
2. Find decimal expansion of  $\frac{17}{8}, \frac{3}{15}, \frac{2}{7}, \frac{50}{3}$ .
3. Find four rational numbers between  $\frac{2}{9}$  and  $\frac{3}{7}$ .
4. Find decimal form of  $\sqrt{23}$  and  $\sqrt{24}$  upto 3 decimal places.
5. Find two Irrational numbers between  $\sqrt{23}$  and  $\sqrt{24}$ .
6. Find one Irrational and one rational number between 2 and  $\sqrt{5}$ .
7. Write two numbers whose decimal expansions are terminating.
8. What can be the maximum number of digits in the repeating block of digits in the decimal expansion of  $\frac{5}{7}$  ?
9. Write two numbers whose decimal expansions are non-terminating non-repeating (non-recurring).
10. Find the value of  $(256)^{0.16} \times (256)^{0.09}$
11. Find two Irrational numbers between 2016 and 2017.

### **Part (B)**

12. Represent  $\frac{-7}{5}$  on the number line.
13. Represent following on number line  
i)  $\sqrt{5}$       ii)  $\sqrt{13}$       iii)  $\sqrt{9.3}$       iv)  $\sqrt{2}$
14. Represent  $3 + \sqrt{2.6}$  on the number line.
15. Insert two Irrational numbers between  $\frac{2}{3}$  and  $\frac{3}{2}$
16. Simplify:  $\frac{\sqrt{5} + \sqrt{3}}{\sqrt{80} + \sqrt{48} - \sqrt{45} - \sqrt{27}}$
17. Find the value of  $[1^3 + 2^3 + 3^3 + 8^2]^{-5/2}$
18. Find the value of x if  $x^{1/2} = (36)^{0.5}$
19. Find the value of x if  $(\sqrt{3})^x = 3^7$

20. If  $2^{5x} \div 2^x = \sqrt[5]{32}$ . Then find the value of x.
21. Evaluate  $a^{x-y} \cdot a^{y-z} \cdot a^{z-x}$ .
22. Simplify  $12^{\frac{2}{5}} \cdot 5^{\frac{2}{5}}$ .
23. Which of the following rational numbers will have a terminating decimal expansion or a non-terminating repeating (recurring) decimal expansion ?
- (i)  $\frac{135}{50}$       (ii)  $\frac{4}{11}$       (iii)  $\frac{8}{7}$       (iv)  $6\frac{3}{8}$
- (v)  $\frac{55}{9}$       (vi)  $\frac{5^2 \times 3^3}{2 \times 5^3 \times 27}$       (vii)  $\frac{51}{60}$
24. Classify the following numbers as terminating decimal or non-terminating recurring decimal or non-terminating non-recurring decimal :
- (i) 0.1666...      (ii) 0.250.....      (iii) 1.01001000100001....  
 (iv) 0.27696      (v) 2.142857142857....      (vi)  $0.\overline{3}$   
 (vii) 0.2359872785...      (viii) 0.48484884848....      (ix) 2.502500250002....  
 (x) 4. $\overline{123456789}$
- Also classify these given numbers as Rational and Irrational numbers.
25. Classify the following numbers as rational or Irrational number :
- (i)  $\sqrt{27}$       (ii)  $\sqrt{36}$       (iii)  $\sqrt{5} \times \sqrt{125}$       (iv)  $2\sqrt{3}$   
 (v)  $\frac{7\sqrt{7}}{\sqrt{343}}$       (vi)  $2 + \sqrt{21}$       (vii)  $5 + 2\sqrt{23} - (\sqrt{25} + \sqrt{92})$   
 (viii)  $\frac{22}{7}$       (ix)  $\pi$       (x)  $\sqrt[3]{27}$
26. Express the following numbers in the form  $\frac{p}{q}$ , where p and q are integers and  $q \neq 0$ .
- (i) 0.0875      (ii) 2.123456789      (iii) 0.181818.....

27. Do as directed :

- (i) Add:  $\sqrt{125} + 2\sqrt{27}$  and  $-5\sqrt{5} - \sqrt{3}$
  - (ii) Add:  $\sqrt{7} - \sqrt{11}$  and  $\sqrt{5} - \sqrt{11} + \sqrt{13}$
  - (iii) Multiply:  $2\sqrt{2}$  by  $5\sqrt{2}$ .
  - (iv) Multiply:  $(-3 + \sqrt{5})$  by 3.
  - (v) Divide:  $7\sqrt{5}$  by  $-14\sqrt{125}$
  - (vi) Divide:  $2\sqrt{216} - 3\sqrt{27}$  by 3.

### **Part (C)**

28. Simplify :

- (i)  $(2\sqrt{2} + 3\sqrt{3})(2\sqrt{2} - 3\sqrt{3})$       (ii)  $(2\sqrt{8} - 3\sqrt{2})^2$   
 (iii)  $(\sqrt{7} + \sqrt{6})^2$       (iv)  $(6 - \sqrt{2})(2 + \sqrt{3})$

29. Evaluate:

- $$(i) \frac{2^{38} + 2^{37} + 2^{36}}{2^{39} + 2^{38} + 2^{37}}$$

30. Find the value of  $a$  if  $\frac{6}{3\sqrt{2} - 2\sqrt{3}} = 3\sqrt{2} - a\sqrt{3}$ .

31. Simplify:  $\left[ 5(8^{1/3} + 27^{1/3})^3 \right]^{1/4}$

32. Simplify :  $\frac{(25)^{3/2} \times (243)^{3/5}}{(16)^{5/4} \times (8)^{4/3}}$

33. If  $5^{2x-1} - (25)^{x-1} = 2500$ , then find the value of x.

## Part (D)

34. Express  $0.6 + 0.\overline{7} + 0.4\overline{7}$  in the form  $\frac{p}{q}$  where p and q are integers and  $q \neq 0$ .

35. Rationalise the denominator of  $\frac{1}{\sqrt{3} + \sqrt{5} + \sqrt{7}}$

36. Find  $a$  and  $b$  if  $\frac{7+3\sqrt{5}}{2+\sqrt{5}} - \frac{7-3\sqrt{5}}{2-\sqrt{5}} = a + b\sqrt{5}$

37. If  $x = (3 - 2\sqrt{2})$ , show that  $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) = \pm 2$

38. If  $xyz = 1$ , then simplify

$$(1+x+y^{-1}) \times (1+y+z^{-1})^{-1} \times (1+z+x^{-1})^{-1}$$

39. Find the value of  $x$  if

(i)  $25^{2x-3} = 5^{2x+3}$

(ii)  $(4)^{2x-1} - (16)^{x-1} = 384$

40. Evaluate :  $\frac{64^{\frac{a}{6}}}{4^a} \times \frac{2^{2a+1}}{2^{a-1}}$

41. Simplify :  $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{a-c}+x^{b-c}}$

42. Simplify :  $\left(\frac{x^a}{x^{-b}}\right)^{a-b} \times \left(\frac{x^b}{x^{-c}}\right)^{b-c} \times \left(\frac{x^c}{x^{-a}}\right)^{c-a}$

43. Show that :

$$\frac{1}{(3-\sqrt{8})} - \frac{1}{(\sqrt{8}-\sqrt{7})} + \frac{1}{(\sqrt{7}-\sqrt{6})} - \frac{1}{(\sqrt{6}-\sqrt{5})} + \frac{1}{(\sqrt{5}-2)} = 5$$

44. If  $a = \frac{\sqrt{7}-\sqrt{6}}{\sqrt{7}+\sqrt{6}}$  and  $b = \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}-\sqrt{6}}$ , then find the value of  $a^2 + b^2 + ab$ .

45. Simplify :  $\frac{2\sqrt{6}}{\sqrt{2}+\sqrt{3}} + \frac{6\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{8\sqrt{3}}{\sqrt{6}+\sqrt{2}}$

46. If  $x = 9 - 4\sqrt{5}$ , then find

(i)  $x + \frac{1}{x}$       (ii)  $x - \frac{1}{x}$       (iii)  $x^2 + \frac{1}{x^2}$       (iv)  $x^2 - \frac{1}{x^2}$

(v)  $x^3 + \frac{1}{x^3}$       (vi)  $x^3 - \frac{1}{x^3}$       (vii)  $\sqrt{x} + \frac{1}{\sqrt{x}}$       (viii)  $\sqrt{x} - \frac{1}{\sqrt{x}}$

(ix)  $x^4 + \frac{1}{x^4}$     (x)  $x^6 + \frac{1}{x^6}$     (xi)  $x + \frac{14}{x}$

47. If  $a = 1 + \sqrt{7}$ , find the value of  $\frac{-6}{9}$

48. If  $p = 5 - 2\sqrt{6}$ , Find  $p^2 + \frac{1}{p^2}$

49. Express  $0.\overline{3178}$  in the form of  $p/q$  where  $p$  and  $q$  are integers and  $q \neq 0$ .

50. If  $\sqrt{2} = 1.414$ , then find the value of  $\sqrt{8} + \sqrt{50} + \sqrt{72} + \sqrt{98}$

51. Find the value of

$$\frac{4}{(216)^{\frac{-2}{3}}} + \frac{1}{(256)^{\frac{-3}{4}}} + \frac{2}{(243)^{\frac{-1}{5}}}$$

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## CHAPTER-1

# NUMBER SYSTEMS

### ANSWERS

- 1)  $\frac{0}{1}, \frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1}$
- 2)  $\frac{17}{8} = 2.125, \frac{3}{15} = 0.2, \frac{2}{7} = 0.\overline{285714}, \frac{50}{3} = 16.\overline{6}$
- 3)  $\frac{15}{63}, \frac{16}{63}, \frac{17}{63}, \frac{18}{63}$  (other answers are possible).
- 4)  $\sqrt{23} = 4.795, \sqrt{24} = 4.898$
- 5) 4.8010010001 ..... , 4.8020020002 ..... , (other answers are possible)
- 6) 2.1, 2.010010001 ..... , (other answers are possible).
- 8) 6
- 10) 4
- 11) 2016.1010010001 ..... ; 2016.2020020002 ..... ; (other answers are possible)
- 15) 0.909009000 ..... ; 1.10100100010000 ..... (other answers are possible)
- 16) 1      17)  $\frac{1}{10^5}$       18) 36      19) 14
- 20)  $x = \frac{1}{4}$       21) 1      22)  $(60)^{2/5}$
- 23) (i) Terminating Decimal      (ii) Non Terminating Repeating Decimal  
(iii) Non-Terminating Repeating Decimal  
(iv) Terminating Decimal      (v) Non-Terminating Repeating Decimal  
(vi) Terminating Decimal      (vii) Terminating Decimal

- 24) (i) Non-Terminating Repeating Decimal (Rational).  
(ii) Terminating Decimal (Rational).  
(iii) Non-Terminating Non-Repeating Decimal (Irrational).  
(iv) Terminating Decimal (Rational)  
(v) Non-Terminating Repeating Decimal (Rational)  
(vi) Non-Terminating Repeating Decimal (Rational)  
(vii) Non-Terminating Non-Repeating Decimal (Irrational)  
(viii) Non-Terminating Non-Repeating Decimal (Irrational)  
(ix) Non-Terminating Non-Repeating Decimal (Irrational)  
(x) Non-Terminating Repeating Decimal (Rational).
25. (i) Irrational      (ii) Rational      (iii) Rational      (iv) Irrational  
(v) Rational      (vi) Irrational      (vii) Rational      (viii) Rational  
(ix) Irrational      (x) Rational
26. (i)  $0.0875 = \frac{7}{80}$       (ii)  $\frac{2123456789}{1000000000}$       (iii)  $\frac{2}{11}$   
(iv)  $\frac{433}{990}$       (v)  $\frac{1643}{450}$
27. (i)  $5\sqrt{3}$       (ii)  $\sqrt{5} - 2\sqrt{11} + \sqrt{7} + \sqrt{13}$       (iii) 20  
(iv)  $-9 + 3\sqrt{5}$       (v)  $-\frac{1}{10}$       (vi)  $4\sqrt{6} - 3\sqrt{3}$
28. (i) -19      (ii) 2      (iii)  $13 + 2\sqrt{42}$   
(iv)  $12 + 6\sqrt{3} - 2\sqrt{2} - \sqrt{6}$
29. (i)  $\frac{1}{2}$       (ii) 2
30.  $a = -2$       31. 5      32.  $\frac{3375}{512}$
33.  $x = 3$       34.  $\frac{167}{90}$

35.  $\frac{1}{59} (9\sqrt{3} + 5\sqrt{5} + \sqrt{7} - 2\sqrt{105})$

36.  $a = 0, b = 2$

38.  $\frac{1}{(1+y+xy)(1+z+yz)(1+x+zx)}$

39. (i)  $\frac{9}{2}$  (ii)  $\frac{11}{4}$

40. 4

44.  $a^2 + b^2 + ab = 675$

45.  $8\sqrt{3} - 14\sqrt{2}$

46. (i) 18

(ii)  $-8\sqrt{5}$

42. 1

(iv)  $-144\sqrt{5}$

(v) 5778

(vi)  $-2584\sqrt{5}$

(vii)  $2\sqrt{5}$

(viii) 4

(ix) 103682

(x) 33385282

(xi)  $8\sqrt{3} - 14\sqrt{2}$

47.  $1-\sqrt{7}$

48. 98

49.  $\frac{635}{1998}$

50. 28.28

51. 214

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