CHAPTER-2

POLYNOMIALS

KEY POINTS

- A Polynomial p(x) in one variable x is an algebraic expression in x of the form $p(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$, where $a_0, a_1, a_2 ..... a_n$ are real numbers and $a_n \neq 0$ are called coefficients and n is a whole numbers.
- The highest power of variable x in a polynomial p(x) is called the degree of the polynomial.
- $a_0, a_1, a_2, ..., a_n$ are respectively the coefficients of $x^0, x, x^2, ... x^n$, and n is called the degree of the polynomial. Each of $a_n x^n, a_{n-1} x^{n-1} ..... a_0$ with $a_n \neq 0$, is called a term of the polynomial p (x)
- A polynomial having one term is called monomial, having two terms called binomial and having three terms called trinomial.
- A polynomial of degree one is called linear polynomial, having degree two is called quadratic polynomial and of degree three is called cubic polynomial.
- For a polynomial p(x) if p(a) = 0 where a is a real number we say that 'a' is a zero of the polynomial.
- If p(x) is any polynomial of degree greater than or equal to 1 and p(x) is divided by a linear polynomial x – a, then the remainder is p(a). This is called remainder theorem.
- If p(x) is a polynomial of degree ≥ 1 and 'a' is any real number then

  (i) (x – a) is a factor of p(x), if p(a) = 0 and

  (ii) p(a) = 0 if (x – a) is a factor of p(x).

  This is called factor theorem.
- A polynomial of degree 'n' can have at most n zeroes.
- Some algebraic identities :–

  (i) $(x+y)^2 = x^2 + 2xy + y^2$

  (ii) $(x-y)^2 = x^2 - 2xy + y^2$

  (iii) $x^2 - y^2 = (x+y)(x-y)$

  (iv) $(x+a)(x+b) = x^2 + (a+b)x + ab$

(v) $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(vi) $(x+y)^3 = x^3 + y^3 + 3xy\,(x+y) = x^3 + y^3 + 3x^2y + 3xy^2$

(vii) $(x-y)^3 = x^3 - y^3 - 3xy\,(x-y) = x^3 - y^3 - 3x^2y + 3xy^2$

(viii) $x^3+y^3 = (x+y)\,(x^2 - xy + y^2)$

ix) $x^3-y^3 = (x-y)\,(x^2+xy + y^2)$

x) $x^3 + y^3 + z^3 - 3xyz = (x+y+z)\,(x^2+y^2+z^2-xy-yz-zx)$

$$= \frac{1}{2}(x+y+z)\left\{(x-y)^2 + (y-z)^2 + (z-x)^2\right\}$$

xi) If $x+y+z = 0$, then $x^3 + y^3 + z^3 = 3xyz$

Part-A

1. Write the coefficient of y^3 in $5y^3 + 2y^2 - y + 5$

2. Find the coefficient of x^2 in $(x^2 - 1)(x - 2)$

3. If $(x - 2)$ is one of the factor of $3x - 2a$, then find the value of a.

4. Find the degree of polynomial $\frac{x^3 + 3x - 1}{5} - \frac{5}{2}x^2 - x^5$

5. If $p(x) = x^3 - 3x^2 + 2x - 3$ find the value of $p(1) + p(-1)$.

6. Find zeros of the polynomial $z^2 - 8$

7. Divident = Divisor × Quotient + _____.

8. Give an example of Trinomial of degree 3.

9. Give one example of each monomial, binomial and quadratic polynomial.

10. Check whether $x = 3$ is a zero of polynomial $x^2 - 3x + x - 3$.

11. Write the degree of the polynomial $\sqrt{7}$

12. If one of the zero of polynomial $3x^2 + 5x + k$ is -1, then find out the value of k.

13. Express $4x^2 - 4x + 1$ as a square of binomial.

Part – B

14. Check whether q(x) is a multiple of r(x) or not.

    If $q(x) = 2x^3 - 11x^2 - 4x + 5$, $r(x) = 2x + 1$

15. Show that $(x - 5)$ is a factor of $x^3 - 3x^2 - 4x - 30$ by Remainder theorom.

16. Evaluate by using suitable identity : $(997)^3$

17. Find the zeroes of the polynomial $p(x) = x(x-2)(x+3)$

18. Find the quotient when $3x^2 - 7x - 6$ is divided by $(x-3)$

19. Factorise $8x^3 + \sqrt{27}\,y^3$.

20. If $p(x) = x + 9$, then find $p(x) + p(-x)$.

21. Find the product without multiplying directly

    106×94

22. IF $36x^2 - b = \left(6x + \frac{1}{5}\right)\left(6x - \frac{1}{5}\right)$ then find the value of b.

23. Expand using suitable identity $(2x - 3y + z)^2$

24. Find the value of $(351)^2 - (350)^2$.

Part – C

25. Factorise : $64a^2 + 96ab + 36b^2$

26. Factorise : $x^3 + 6x^2 + 11x + 6$

27. If $x^2 + y^2 = 49$ and $x - y = 3$, then find the value of $x^3 - y^3$.

28. Simplify : $(5a-2b)(25a^2 + 10ab + 4b^2) - (2a+5b)(4a^2 - 10ab + 25b^2)$

29. Find the sum of remainders when $x^3 - 3x^2 + 4x - 4$ is divided by $(x-1)$ and $(x+2)$.

30. Find the product $\left(p - \frac{1}{p}\right)\left(p + \frac{1}{p}\right)\left(p^2 + \frac{1}{p^2}\right)\left(p^4 + \frac{1}{p^4}\right)$

31. Factorise : $7\sqrt{2}\,k^2 - 10k - 4\sqrt{2}$.

32. Simplify : $(3x-4y)^3 - (3x+4y)^3$

33. Expand : $\left(\frac{1}{2}x - \frac{1}{4}y + 2\right)^2$ using suitable identity.

34. Simplify : $(x+y+z)^2 - (x-y-z)^2$.

Part – D

35. Factorise : $125x^3 + 8y^3 + z^3 - 30xyz$.

36. $x + 2$ is a factor of polynomial $ax^3 + bx^2 + x - 2$ and the remainder 4 is obtained by dividing this polynomial by $(x - 2)$. Find the value of a and b.

37. Check whether

    $p(t) = 6t^3 + 3t^2 + 3t + 18$ is a multiple of $(2t + 3)$.

38. Find the value of k if $(x + k)$ is a factor of the polynomial $x^3 + kx^2 - 2x + k + 4$ and factorise $x^4 - x$.

39. If $(x - 3)$ and $\left(x - \frac{1}{3}\right)$ are factors of the polynomial $px^2 + 3x + r$, show that $p = r$.

40. (i) Using Identity, find the value of $(-7)^3 + (5)^3 + (2)^3$.

    (ii) Find dimension of cube whose volume is given by expression $4x^2 + 14x + 6$

41. Give possible expression for the length and breadth of each of the following rectangles if.

    (i) Area = $(x^2 + 5\sqrt{5}x + 30)$ sq. unit.

    (ii) Area = $(24x^2 - 26x - 8)$ sq. unit.

42. A literacy compaign was organised by Class IX girl students under NSS. Students made $(x - 5)$ rows and $(3x - 4)$ columns for the rally.

    (a) Write the total number of students in the form of polynomial.

    (b) Which values of students are depicted here?

43. Under tree plantation programme students of Class IX planted total $(3x^2 - 4x - 4)$ trees in school.

    (i) If total number of students in the class are $(x - 2)$ then find out number of trees planted by each student. (Assuming each student planted equal number of trees).

    (ii) What values of students are exhibited here?

44. If $a + b + c = 0$, find the value of

$$\frac{(b+c)^2}{bc} + \frac{(c+a)^2}{ca} + \frac{(a+b)^2}{ab}$$

45. Simplify :

$$\frac{(a^2-b^2)^3 + (b^2-c^2)^3 + (c^2-a^2)^3}{(a-b)^3 + (b-c)^3 + (c-a)^3}$$

46. Factorise :

$$(2a-b-c)^3 + (2b-c-a)^3 + (2c-a-b)^3$$

47. If the polynomial $4x^3-16x^2+ax+7$ is exactly divisible by $x-1$, then find the value of a. Hence factorise the polynomial.

CHAPTER-2

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ANSWERS

1. 5
2. –2
3. $a = 3$
4. 5
5. –12
6. $+\sqrt{8}, -\sqrt{8}$
7. Remainder
8. $x^3 - 3x^2 + 2$ or any other example
9. $2x, 2x^2 + 3, x^2 + 2x - 3$ or any other examples
10. Yes
11. Degree = 0
12. $k = 2$
13. $(2x-1)^2$
14. No.
15. Hint put $x = 5$
16. 991026973
17. 0, 2, –3
18. $3x + 2$
19. $\left(2x + \sqrt{3}y\right)\left(4x^2 - 2\sqrt{3}xy + 3y^2\right)$
20. 18
21. Hint $(100 + 6)(100 - 6)$
22. $\frac{1}{25}$
23. $4x^2 + 9y^2 + z^2 - 12xy - 6yz + 4xz$
24. 701
25. $(8a + 6b)^2$
26. $(x + 1)(x + 2)(x + 3)$
27. 207
28. $117a^3 - 133b^3$
29. –34
30. $p^8 - \frac{1}{p^8}$
31. $\left(k - \sqrt{2}\right)\left(7\sqrt{2}k + 4\right)$
32. $-8y(16y^2 + 27x^2)$ or $-128y^3 - 216x^2y$
33. $\frac{x^2}{4} + \frac{y^2}{16} + 4 - \frac{1}{4}xy - y + 2x$
34. $4xy + 4zx$
35. $(5x + 2y + z)(25x^2 + 4y^2 + z^2 - 10xy - 2yz - 5zx)$
36. $a = 0, b = 2$
37. Yes
38. $k = \frac{4}{3}, x(x-1)\left(x^2 + x + 1\right)$
40. (i) –210; (ii) $2, (x + 3), (2x + 1)$

41. (i) $(x + 2\sqrt{5}), (x + 3\sqrt{5})$ (ii) $(4x + 1), (6x - 8)$

42. (a) $3x^2 - 19x + 20$

(b) Social responsibility, Empathy, etc.

43. (i) $(3x + 2)$

(ii) Scientific attitude, Dutiful, Environment awareness, Social values

44. 3

45. $(a+b)(b+c)(c+a)$

46. $3(2a-b-c)(2b-c-a)(2c-a-b)$

47. $a = 5, (x-1)(2x+1)(2x-7)$